Mathematics is beautiful by Heinz Klaus Strick, pp. 366, £24.99 (paper), ISBN 978-3-662-62688-7, £19.99 (eBook) ISBN 978-3-662-62689- 4, Springer Verlag (2021)

The subtitle of this book is 'Suggestions for people between 9 and 99 years to look at and explore'. The author has used the material of the book in his work as a schoolteacher and head teacher in Germany to 'loosen up my lessons', but the age span suggests a wider audience which might be attracted by the hundreds of excellent colour illustrations: there is hardly a page without at least one. Thus the choice of material is partly governed by its visual appeal, but there is plenty of algebra in evidence too so this is not a coffee-table book for casual reading. The level of mathematical maturity needed to follow the material varies greatly through the book and the author quotes without proof and uses some moderately hard theorems when they lead to interesting applications—for example 'Descartes' Theorem' about the curvatures (reciprocal radii) k_i of four circles each of which is tangent to the other three: $2\left(k_1^2 + k_2^2 + k_3^2 + k_4^2\right) = \left(k_1 + k_2 + k_3 + k_4\right)^2$. Each section concludes with 'suggestions for reflection and investigations' which take the material further or ask leading questions to enhance understanding. The references are nearly all to websites such as Wikipedia and Wolfram Mathworld, though a few printed sources are listed at the end of the book.

Here are a just a few of the topics covered in 17 chapters; I hope this gives an idea of the wide range of the material included. 'Stars' which are obtained from a regular n-sided polygon, by joining every kth vertex for a fixed k, are the basis for the first chapter, with a detailed treatment including the number of components of a star, the number of distinct stars for a given n, the lengths of edges, the total length of a star and the associated angles, regular n-sided figures in the complex plane, a discussion of de Moivre's theorem and solutions of $z^n = 1$. There is also an application to setting up tournament schedules where each team plays each of the others.

Dissection of rectangles into disjoint squares is linked to Fibonacci numbers, the Euclidean algorithm and continued fractions for rational numbers and for square roots of integers. A different chapter covers dissection of rectangles into squares of different sizes, including the famous minimal example of Duijvestijn in 1978 dissecting a square into 21 squares of different sizes. There is an opportunity here to use simultaneous linear equations to determine the sizes of the squares, given the dissection, and also an application to electrical circuits and Kirchhoff's laws.

There is a chapter on areas and perimeters of figures drawn on a lattice, first made up of lattice squares but then more generally, leading to Pick's theorem, that the area of a lattice polygon equals the number of interior lattice points plus half the number of boundary lattice points, minus 1. The theorem is proved in a sequence of elementary steps, more or less following a standard pattern. The next chapter is about rolling pairs of standard dice, constructing histograms for the sum of spots on the upper faces, but also including a pair of 'Sicherman dice' which have different arrangements of spots but the same distribution of scores. There are also discussions of rolling more than two dice, 'wheel of fortune' models and generating functions for probabilities, and even probability distributions for rolling other Platonic solids, a simple example of the central limit theorem and Markov chains. The final chapter of the book is about Pythagoras's theorem and the many dissection proofs. In fact dissection and tessellation are a major theme in the book, including dissection of regular polygons into equal-area parts, leading to a discussion of geometric series, and tessellation of regular polygons by rhombi.

The book is translated from German, and according to a note on page iv of the

front matter this was done by machine using the service DeepL.com. This has resulted in a generally clear, but rather 'wooden'—as opposed to 'lively'—style, with some peculiar sentences which need to be read twice to realise there is a redundant word. There is a useful index and, considering the huge number of coloured illustrations, the price is not exorbitant. A very interesting book with lots of useful material for enrichment at many levels (and why stop at 99?).

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